Paper Reference(s)

**6690/01**

# Edexcel GCE

## Decision Mathematics D2

## Advanced/Advanced Subsidiary

Thursday 6 June 2013 – Morning

Time: 1 hour 30 minutes

**Materials required for examination**
Nil

**Items included with question papers**
D2 Answer Book

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.
In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
Do not return the question paper with the answer book.

### Information for Candidates

Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 8 pages in this question paper. The answer book has 16 pages. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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**Write your answers in the D2 answer book for this paper.**

**1.**

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 15 | 19 | 25 | 20 |
| B | 15 | – | 15 | 15 | 25 |
| C | 19 | 15 | – | 22 | 11 |
| D | 25 | 15 | 22 | – | 18 |
| E | 20 | 25 | 11 | 18 | – |

The table shows the least distances, in km, between five hiding places, A, B, C, D and E.

Agent Goodie has to leave a secret message in each of the hiding places. He will start and finish at A, and wishes to minimise the total distance travelled.

(a) Use Prim's algorithm to find a minimum spanning tree for this network. Make your order of arc selection clear.

**(2)**

(b) Use your answer to part (a) to determine an initial upper bound for the length of Agent Goodie's route.

**(1)**

(c) Show that there are two nearest neighbour routes which start from A. State these routes and their lengths.

**(3)**

(d) State the better upper bound from your answers to (b) and (c).

**(1)**

(e) Starting by deleting B, and all of its arcs, find a lower bound for the length of Agent Goodie's route.

**(4)**

(f) Consider your answers to (d) and (e) and hence state an optimal route.

**(1)**

**(Total 12 marks)**

**2.** The table shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of three demand points, 1, 2 and 3. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | 10 | 11 | 20 | 18 |
| B | 15 | 7 | 13 | 14 |
| C | 24 | 15 | 12 | 21 |
| D | 9 | 21 | 18 | 12 |
| Demand | 27 | 18 | 20 | |

(a) Use the north-west corner method to obtain an initial solution.

**(1)**

(b) Taking D1 as the entering cell, use the stepping stone method to find an improved solution. Make your route clear.

**(2)**

(c) Perform one further iteration of the stepping stone method to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, route, entering cell and exiting cell.

**(4)**

(d) Determine whether your current solution is optimal, giving a reason for your answer.

**(3)**

**(Total 10 marks)**

**3.**

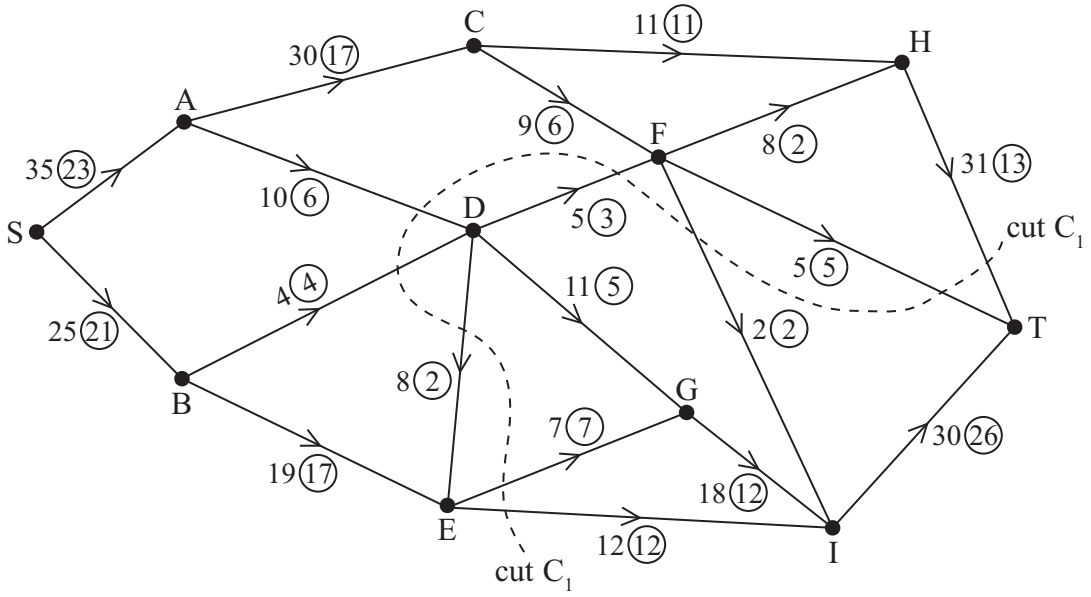



**Figure 1**

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

**(1)**

(b) State the capacity of cut $C_1$.

**(1)**

The labelling procedure has been used and the result drawn on Diagram 1 in the answer book.

(c) Use Diagram 1 to find the maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

**(4)**

(d) Draw a maximum flow pattern on Diagram 2 in your answer book.

**(2)**

(e) Prove that the flow shown in (d) is maximal.

**(2)**

**(Total 10 marks)**

**4.** A two-person zero-sum game is represented by the following pay-off matrix for player A.

| | B plays 1 | B plays 2 | B plays 3 |
|---|---|---|---|
| A plays 1 | 5 | 4 | –6 |
| A plays 2 | –1 | –2 | 3 |
| A plays 3 | 1 | –1 | 2 |

(a) Reduce the game so that player B has only two possible actions.

**(1)**

(b) Write down the reduced pay-off matrix **for player B**.

**(2)**

(c) Find the best strategy for player B and the value of the game to him.

**(8)**

**(Total 11 marks)**

**5.** In solving a three-variable maximising linear programming problem, the following tableau was obtained after the first iteration.

| Basic variable | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
|---|---|---|---|---|---|---|---|
| $r$ | –1 | 2 | 0 | 1 | 0 | 1 | 8 |
| $s$ | –1 | 3 | 0 | 0 | 1 | 1 | 22 |
| $z$ | –2 | 1 | 1 | 0 | 0 | 1 | 11 |
| $P$ | 2 | –5 | 0 | 0 | 0 | $\frac{1}{2}$ | 15 |

(a) State which variable was increased first, giving a reason for your answer.

**(1)**

(b) Solve this linear programming problem. Make your method clear by stating the row operations you use.

**(8)**

(c) State the final value of the objective function and the final values of each variable.

**(2)**

**(Total 11 marks)**

**6.** Three workers, Harriet, Jason and Katherine, are to be assigned to three tasks, 1, 2 and 3. Each worker must be assigned to just one task and each task must be done by just one worker.

The amount each person would earn, in pounds, while assigned to each task is shown in the table below.

| | Task 1 | Task 2 | Task 3 |
|---|---|---|---|
| Harriet | 251 | 243 | 257 |
| Jason | 244 | 247 | 255 |
| Katherine | 249 | 252 | 246 |

The total income is to be maximised.

(a) Modify the table so it can be used to find the maximum income.

**(1)**

(b) Formulate the above situation as a linear programming problem. You must define your decision variables and make your objective function and constraints clear.

**(7)**

**(Total 8 marks)**

**7.** Nigel has a business renting out his fleet of bicycles to tourists.

At the start of each year Nigel must decide on one of two actions:

- Keep his fleet of bicycles, incurring maintenance costs.
- Replace his fleet of bicycles.

The cost of keeping the fleet of bicycles,
the cost of replacing the fleet of bicycles
and the annual income are dependent on the age of the fleet of bicycles.
Table 1 shows these amounts, in £1000s.

| Age of fleet of bicycles | new | 1 year old | 2 years old | 3 years old | 4 years old |
|---|---|---|---|---|---|
| Cost of keeping (£1000s) | 0 | 1 | 2 | 3 | 8 |
| Cost of replacing (£1000s) | – | 7 | 8 | 9 | 10 |
| Income (£1000s) | 11 | 8 | 5 | 2 | 0 |

**Table 1**

Nigel has a new fleet of bicycles now and wishes to maximise his total profit over the next four years.

He is planning to sell his business at the end of the fourth year.

The amount Nigel will receive will depend on the age of his fleet of bicycles.

These amounts, in £1000s, are shown in Table 2.

| Age of fleet of bicycles at end of 4th year | 1 year old | 2 years old | 3 years old | 4 years old |
|---|---|---|---|---|
| Amount received at end of 4th year (£1000s) | 6 | 4 | 2 | 1 |

**Table 2**

Complete the table in the answer book to determine Nigel's best strategy to maximise his total profit over the next four years. You must state the action he should take each year (keep or replace) and his total profit.

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

| Centre No. | | | | | | Paper Reference | | | | | | | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 | 6 | 9 | 0 | / | 0 | 1 | Signature | |

Paper Reference(s)

**6690/01**

# Edexcel GCE

## Decision Mathematics D2

## Advanced/Advanced Subsidiary

Thursday 6 June 2013 – Morning

## Answer Book

**Do not return the question paper with the answer book**

Examiner's use only

| | | |
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Team Leader's use only

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**1.** (a)

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 15 | 19 | 25 | 20 |
| B | 15 | – | 15 | 15 | 25 |
| C | 19 | 15 | – | 22 | 11 |
| D | 25 | 15 | 22 | – | 18 |
| E | 20 | 25 | 11 | 18 | – |

(b)

(c)


| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 15 | 19 | 25 | 20 |
| B | 15 | – | 15 | 15 | 25 |
| C | 19 | 15 | – | 22 | 11 |
| D | 25 | 15 | 22 | – | 18 |
| E | 20 | 25 | 11 | 18 | – |

Leave blank

**Question 1 continued**

(d)

(e)

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 15 | 19 | 25 | 20 |
| B | 15 | – | 15 | 15 | 25 |
| C | 19 | 15 | – | 22 | 11 |
| D | 25 | 15 | 22 | – | 18 |
| E | 20 | 25 | 11 | 18 | – |

(f)

**Q1**

**(Total 12 marks)**

Leave blank

**2.**

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | 10 | 11 | 20 | 18 |
| B | 15 | 7 | 13 | 14 |
| C | 24 | 15 | 12 | 21 |
| D | 9 | 21 | 18 | 12 |
| Demand | 27 | 18 | 20 | |

(a)

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |


| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |

Leave blank

**Question 2 continued**

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |


| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |

| | 1 | 2 | 3 | Supply |
|---|---|---|---|---|
| A | | | | 18 |
| B | | | | 14 |
| C | | | | 21 |
| D | | | | 12 |
| Demand | 27 | 18 | 20 | |


**(Total 10 marks)**

**Q2**

**3.**

(a) Value of initial flow _____ (b) Capacity of cut _____

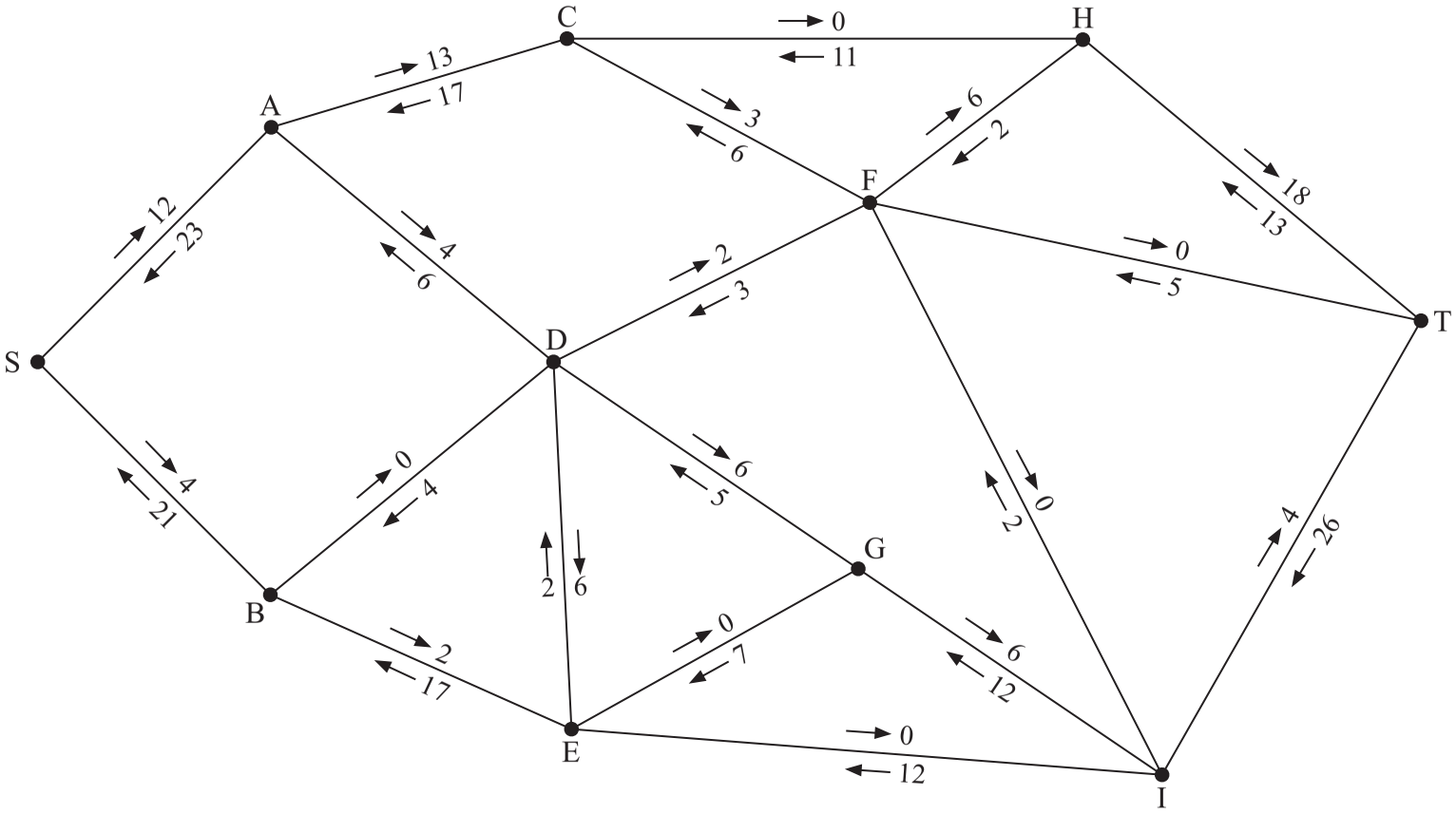



**Diagram 1**

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**Question 3 continued**

(c)

(d)

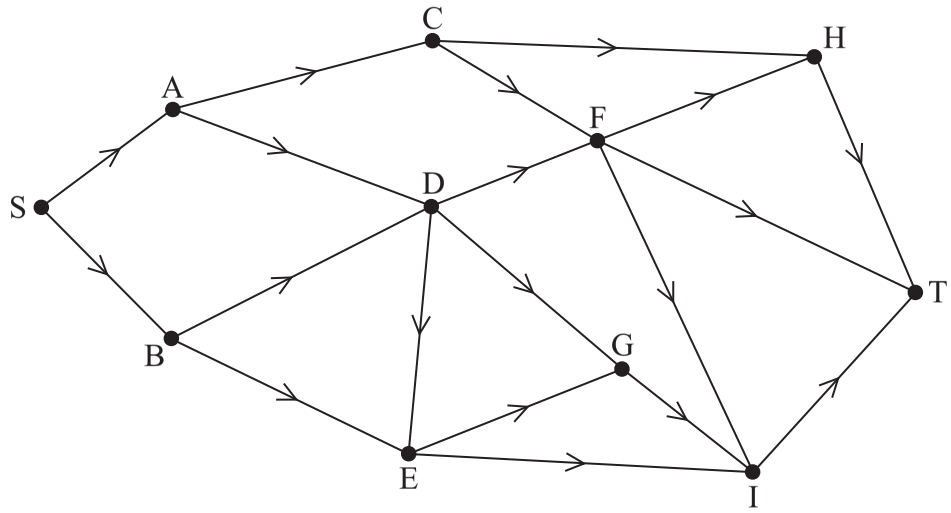



**Diagram 2**

(e)

**(Total 10 marks)**

**Q3**

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Leave blank

**4.**

| | B plays 1 | B plays 2 | B plays 3 |
|---|---|---|---|
| A plays 1 | 5 | 4 | –6 |
| A plays 2 | –1 | –2 | 3 |
| A plays 3 | 1 | –1 | 2 |

Leave blank

**Question 4 continued**

**Q4**

**(Total 11 marks)**

Leave blank

**5.** (a)

| b.v. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value |
|---|---|---|---|---|---|---|---|
| $r$ | –1 | 2 | 0 | 1 | 0 | 1 | 8 |
| $s$ | –1 | 3 | 0 | 0 | 1 | 1 | 22 |
| $z$ | –2 | 1 | 1 | 0 | 0 | 1 | 11 |
| $P$ | 2 | –5 | 0 | 0 | 0 | $\frac{1}{2}$ | 15 |

(b) *You may not need to use all of these tableaux*

| b.v. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value | Row Ops |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| $P$ | | | | | | | | |

| b.v. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value | Row Ops |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
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| | | | | | | | | |
| $P$ | | | | | | | | |

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**Question 5 continued**

| b.v. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value | Row Ops |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| $P$ | | | | | | | | |

| b.v. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value | Row Ops |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| $P$ | | | | | | | | |

| b.v. | $x$ | $y$ | $z$ | $r$ | $s$ | $t$ | Value | Row Ops |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| $P$ | | | | | | | | |

(c)

**Q5**

**(Total 11 marks)**

Leave blank

**6.**

| | Task 1 | Task 2 | Task 3 |
|---|---|---|---|
| Harriet | 251 | 243 | 257 |
| Jason | 244 | 247 | 255 |
| Katherine | 249 | 252 | 246 |

Leave blank

**Question 6 continued**

**Q6**

**(Total 8 marks)**

Leave blank

**7.** *You may not need to use all the rows in this table*

| **Stage** | **State** (Age of fleet of bicycles at start of year) | **Action** (Keep or Replace) | **Dest.** (Age of fleet of bicycles at end of year) | **Value** (Profit) |
|---|---|---|---|---|
| End of Year 4 | 4 | (Sell) | - | 1* |
| | 3 | (Sell) | - | 2* |
| | 2 | (Sell) | - | 4* |
| | 1 | (Sell) | - | 6* |
| Year 4 | 3 | Keep | 4 | 1 + 2 – 3 = 0 |
| | | Replace | 1 | 6 + 11 – 9 = 8* |
| | 2 | | | |
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**Question 7 continued**

| **Stage** | **State** (Age of fleet of bicycles at start of year) | **Action** (Keep or Replace) | **Dest.** (Age of fleet of bicycles at end of year) | **Value** (Profit) |
|---|---|---|---|---|
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| Start of year | First year | Second year | Third year | Fourth year |
|---|---|---|---|---|
| Keep/Replace? | | | | |

Total profit

**Q7**

**(Total 13 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

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